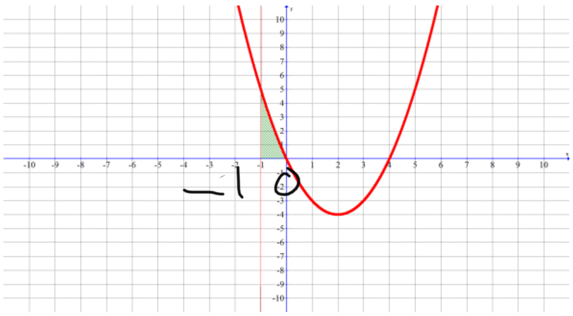
Section 5.5 Solutions

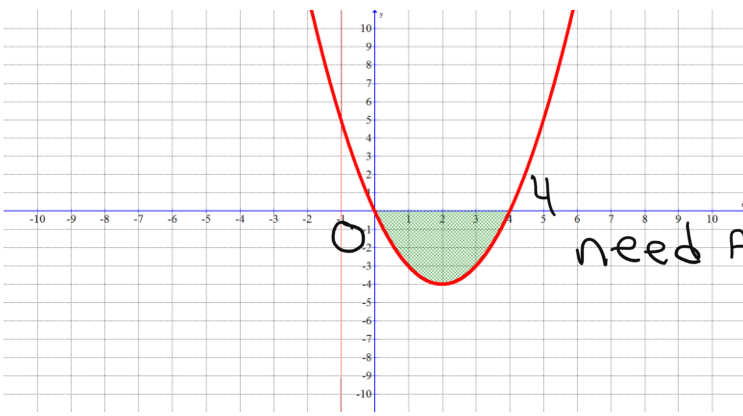
1) The function $f(x) = x^2 - 4x$ *is graphed below.*

1a) Use integration on your calculator to determine the area shaded below between x = -1 and x = 0



Answer $\int_{-1}^{0}(x^2 - 4x)dx = 2.33$

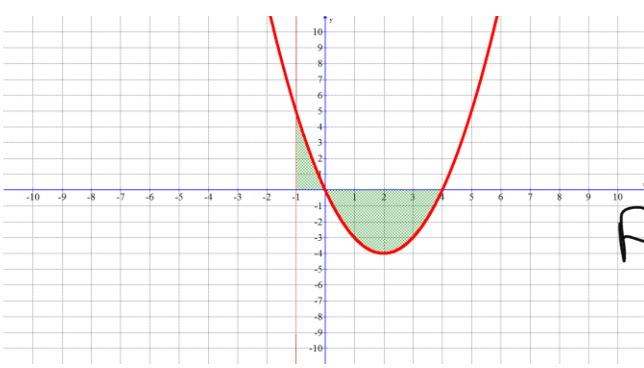
1b) Use integration on your calculator to determine the area shaded below between x = 0 and x = 4



need Absolute Value
Since beneath X-Axis

Answer $\left|\int_{0}^{4}(x^2 - 4x)dx\right| = |-10.67| = 10.67$

1c) Use integration on your calculator to determine the area shaded below between x = -1 and x = 4

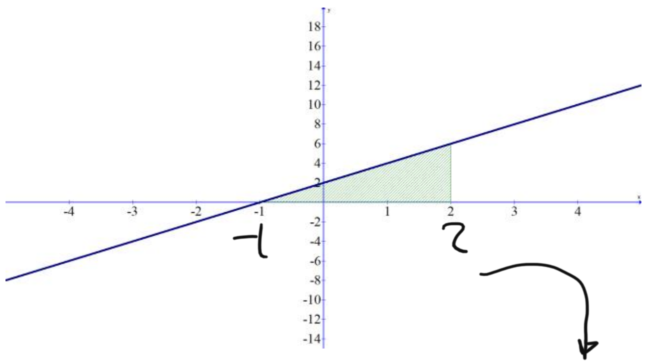


Add answers to part a/b

answer $= \int_{-1}^{0}(x^2 - 4x)dx + \left|\int_{0}^{4}(x^2 - 4x)dx\right| = 2.33 + 10.67 = 13$

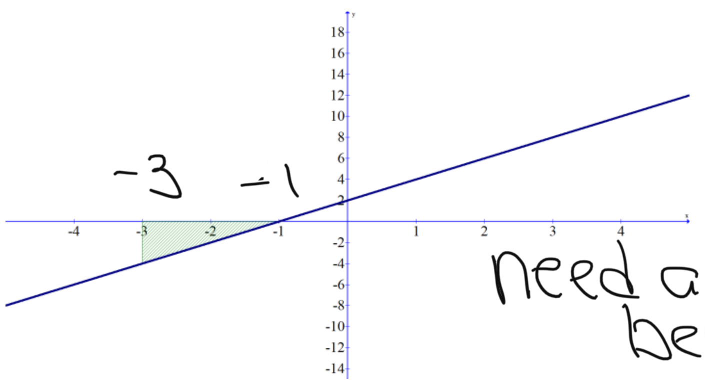
3) The function $f(x) = 2x + 2$ is graphed below

3a) Use integration on your calculator to determine the area shaded below between x = -1 and x = 2



$$answer \int_{-1}^{2} (2x + 2)dx = 9$$

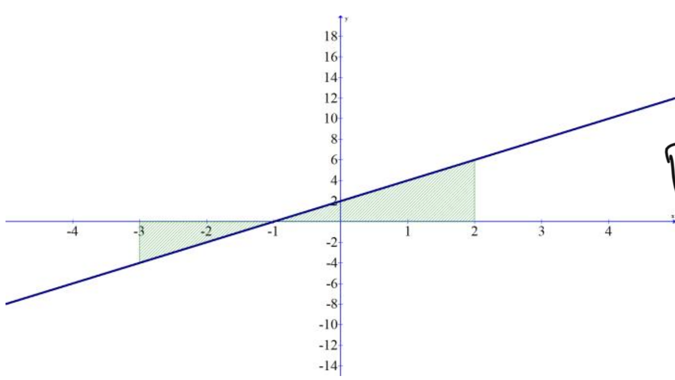
3b) Use integration on your calculator to determine the area shaded below between x = -3 and x = -1



need absolute value since beneath x-axis

$$answer \left| \int_{-3}^{-1} (2x + 2)dx \right| = |-4| = 4$$

3c) Use integration on your calculator to determine the area shaded below between x = -3 and x = 2

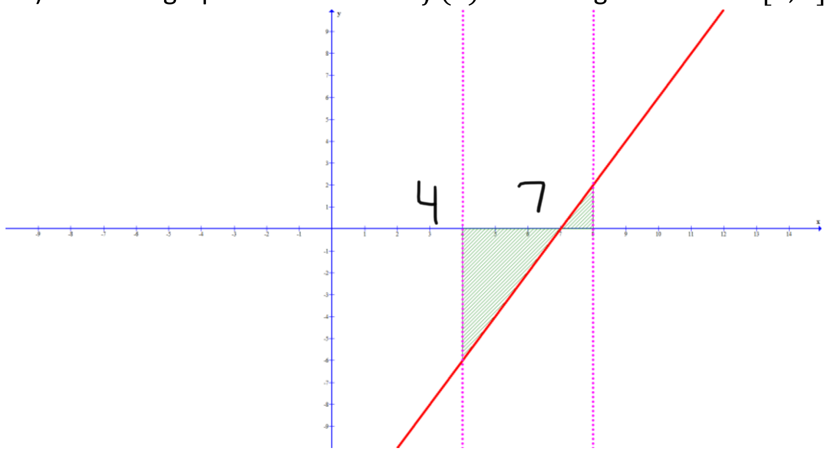


Add answers to a/b

$$answer \int_{-1}^{2} (2x + 2)dx + \left| \int_{-3}^{-1} (2x + 2)dx \right| = 9 + 4 = 13$$

5) $f(x) = 2x - 14;\ \ [4,8]$

5a) Sketch a graph of the function $f(x)$ over the given interval $[a, b]$.



5b) Find any x-intercept within the interval $[a, b]$.

$$2x - 14 = 0$$
$$+14 \ +14$$
$$\frac{2x}{2} = \frac{14}{2}$$
$$x = 7$$

$answer$: $(7,0)$

5c) Find the area between the x-axis and f(x) over the interval $[a, b]$ using definite integrals.

Below the x-axis: $\left|\int_4^7 (2x - 14)\right| dx = |-9| = 9$

$$\left|\int_4^7 (2x-14)dx\right|$$
$$= \left|\int_4^7 2x\,dx - \int_4^7 14\,dx\right|$$
$$= \left|2\int_4^7 x\,dx - \int_4^7 14\,dx\right|$$
$$= \left|2 \cdot \tfrac{1}{2}x^2 - 14x\right|$$
$$= \left| x^2 - 14x \Big|_4^7 \right|$$
$$7^2 - 14(7) = -49$$
$$4^2 - 14(4) = -40$$
$$\left|-49 - (-40)\right|$$
$$|-9| = 9$$

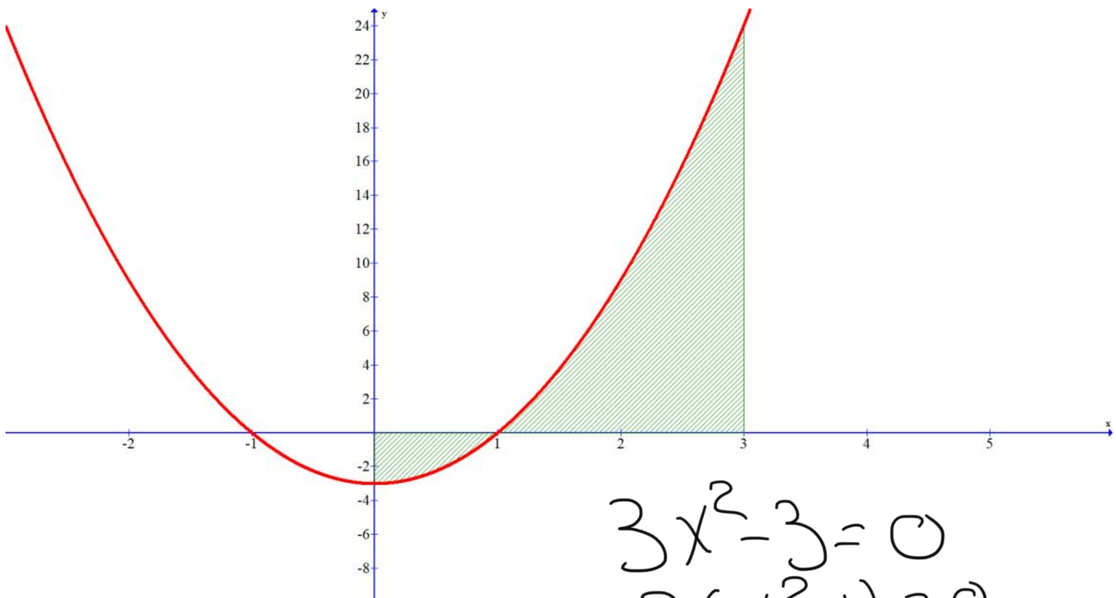
$above\ the\ x - axis$: $\int_7^8 (2x - 14)dx = 1$

$$8^2 - 14(8) = -48$$
$$7^2 - 14(7) = -49$$
$$x^2 - 14x \Big|_7^8$$
$$= -48 - (-49)$$
$$= -48 + 49$$
$$= 1$$

Answer total shaded area $9 + 1 = 10$

7) $f(x) = \ 3x^2 - 3;\ \ [0,3]$

7a) Sketch a graph of the function $f(x)$ over the given interval $[a, b]$.



$3x^2-3=0$

$3(x^2-1)=0$

$3(x+1)(x-1)=0$

7b) Find any x-intercept within the interval $[a, b]$.

$(1,0)$

$3=0$ NO SOL $\quad x+1=0$, $x=-1$ OUTSIDE INTERVAL $\quad x-1=0$, $x=1$

7c) Find the area between the x-axis and f(x) over the interval $[a, b]$ using definite integrals.

Below the x-axis: $\left|\int_0^1 (3x^2-3)\right| dx \ = |-2| = 2$

$\left| x^3-3x \Big|_0^1 \right| = |-2-0| = |-2| =$

$1^3-3(1)=-2$

$0^3-3(0)=0$

$\int_1^3 (3x^2-3)dx$

$= \int_1^3 3x^2 dx - \int 3dx$

$= 3\int_1^3 x^2 dx - \int 3dx$

$= 3\cdot\frac{1}{3}x^3 - 3x$

$= x^3-3x \Big|_1^3$

$= 18-(-2)$

$= 20$

$3^3-3(3)=18$

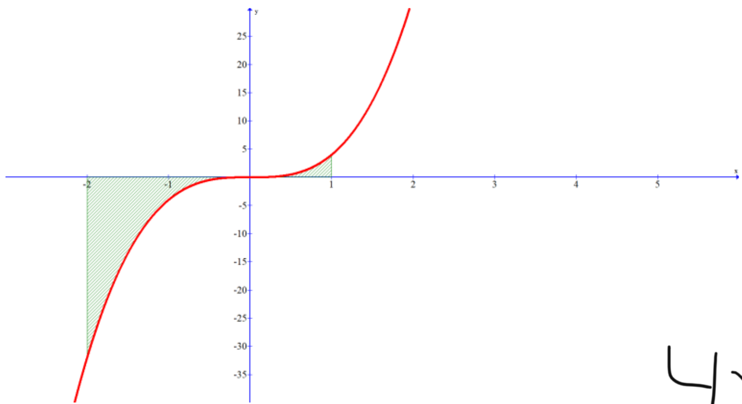
$1^3-3(1)=-2$

above the x − axis: $\int_1^3 (3x^2-3)dx = 20$

Answer: $|-2| + 20 = 2 + 20 = 22$

9) $f(x) = 4x^3$; $[-2,1]$

9a) Sketch a graph of the function $f(x)$ over the given interval $[a, b]$.



$$\frac{4x^3}{4} = \frac{0}{4}$$

$$\sqrt[3]{x^3} = \sqrt[3]{0}$$

$$x = 0$$

9b) Find any x-intercept within the interval $[a, b]$.

$(0,0)$

9c) Find the area between the x-axis and f(x) over the interval $[a, b]$ using definite integrals

Below the x-axis: $\left|\int_{-2}^{0}(4x^3)\right| dx = |-16| = 16$

$$\left| x^4 \Big|_{-2}^{0} \right|$$

$$|0^4-(-2)^4| = |0-16| = |-16| = 16$$

above the x − *axis*: $\int_0^1 (4x^3)dx = 1$ →

$$\int_0^1 4x^3\,dx$$

$$= 4\int x^3\,dx$$

$$= 4\cdot\frac{1}{4}x^4 \Big|_0^1$$

$$= x^4 \Big|_0^1$$

$$= 1^4 - 0^4$$

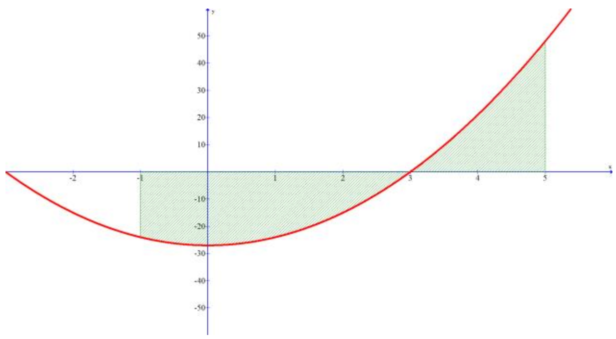
$$= 1 - 0$$

$$= 1$$

Answer: $|-16| + 1 = 16 + 1 = 17$

11) $f(x) = 3x^2 - 27$; $[-1,5]$

11a) Sketch a graph of the function $f(x)$ over the given interval $[a, b]$.



$3x^2 - 27 = 0$

$3(x^2 - 9) = 0$

$3(x+3)(x-3) = 0$

11b) Find any x-intercept within the interval $[a, b]$.

$(3,0)$

$3 = 0$ no sol

$x + 3 = 0$ $\quad x = -3$ NOT in interval

$x - 3 = 0$ $\quad x = 3$

11c) Find the area between the x-axis and f(x) over the interval $[a, b]$ using definite integrals

Below the x-axis: $\left|\int_{-1}^{3}(3x^2 - 27)\right| dx = |-80| = 80$

$\left| x^3 - 27x \Big|_{-1}^{3} \right| = |-54 - 26| = |-80| = 80$

$(3)^3 - 27(3) = -54$

$(-1)^3 - 27(-1) = 26$

above the x − axis: $\int_3^5 (3x^2 - 27)dx = 44$

$\int_3^5 3x^2 dx - \int_3^5 27 dx$

$= 3\int_3^5 x^2 dx - \int_3^5 27 dx$

$= 3 \cdot \frac{1}{3} x^3 - 27x \Big|_3^5$

$= x^3 - 27x \Big|_3^5$

Answer: $|-80| + 44 = 80 + 44 = 124$

$(5)^3 - 27(5) = -10$

$3^3 - 27(3) = -54$

$= -10 - (-54)$

$= 44$